Exact analysis of the combined data of SNO and Super-Kamiokande

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Abstract

Comparison of solar-neutrino signals in SNO [1] and Super-Kamiokande (SK) [2] detectors results in discovery of $\nu_e \to \nu_{\mu,\tau}$ oscillations at level $3.1-3.3\,\sigma$ [1]. This comparison involves the assumption of neutrino spectrum and a choice for the thresholds of detection in both experiments. In this Letter we obtain an exact formula for the comparison of the signals which is valid for arbitrary spectra and thresholds. We find that the no-oscillation hypothesis is excluded at $3.3\,\sigma$. If the energy-dependent component of the survival probability for electron neutrinos is small as compared with the average value, i.e. in the case of small distortion of the observed spectrum, the oscillation hypothesis can also be tested to similar accuracy. The oscillation to sterile neutrino only, is excluded at 3.3σ level, and oscillation to active neutrinos is confirmed at the same level, though with some reservations.

Comparison of Charged Current (CC) solar-neutrino signal in SNO [1] with Elastic Scattering (ES) signal in Super-Kamiokande (SK) [2] has revealed the presence of signal from $\nu_{\mu,\tau}$ neutrinos in SK detector, which evidences for neutrino oscillation. At present this proof of oscillation exists at level $3.1-3.3\,\sigma$, being limited mostly by systematic errors and uncertainties in calculations of CC cross-section: $\nu_e + d \to p + p + e^-$ (for the recent discussion of the latter and the calculations of radiative corrections to the cross-section see Ref. [3]). The basic idea of extracting the signal from another active neutrino component $\nu_a = \nu_{\mu,\tau}$ comes from the fact that in CC scattering in SNO experiment the flux of ν_e neutrinos is measured, while the signal in SK is provided by all active neutrinos including ν_e. SNO gives the total flux of ^{8}B electron neutrinos, *assuming the standard spectrum* [4] as $\phi^{CC}_{SNO} = 1.75 \pm 0.148 \times 10^6$ cm^{-2}s^{-1}. Here and everywhere below we conservatively use the upper systematic errors and sum different errors quadratically. The total ^{8}B neutrino flux detected by SK (*also assuming the standard flux*) is $\phi^{ES}_{SK} = 2.32 \pm 0.085 \times 10^6$ cm^{-2}s^{-1}. If one assumes that only electron and sterile neutrinos are present, the flux above is the flux of ν_e neutrinos. If to allow the presence of other active neutrinos ν_a, the flux above is $\phi_{\nu_e} + 0.154\phi_a$, where 0.154 is the ratio of cross-sections $\nu_a e$ and $\nu_e e$. In any case the $3.3\,\sigma$ excess $\phi^{ES}_{SK} - \phi^{CC}_{SNO} = 0.57 \pm 0.17 \times 10^6$ cm^{-2}s^{-1} [1] signals about presence of another component of active ν_a neutrinos. Note, that both fluxes are obtained assuming the standard (SSM) flux and using for their calculations the different thresholds for electron detection. Adjusting the thresholds, the SNO collaboration arrives at $3.1\,\sigma$ flux difference $0.53 \pm 0.17 \times 10^6$ cm^{-2}s^{-1} [1].

An interesting and effective method of comparison of the signals from SNO and SK was suggested in Refs.([5]). It is based on the demonstrated property that response functions for SNO and SK are approximately equal at the appropriate choice of SNO and SK thresholds.

This method has been further refined in Ref. [6]. Choosing the appropriate thresholds the authors verified the presence of ocillation to active neutrinos at level $3.1\ \sigma$. Among other interesting results the authors have obtained the value of f_B, which characterises possible deviation of boron neutrino flux from the SSM prediction, equal to $1.03^{+0.50}_{-0.58}$.

In Ref. [7] it was demonstrated that in the framework of the Unified Approach [8] the above results correspond to the probability of $\nu_e \to \nu_{\mu,\tau}$ to be larger than zero at 99.89% CL.

Including the results of other solar neutrino experiments and astrophysical information about neutrino production increase considerably the significance of discovery of neutrino oscillation [9].

In this Letter we shall derive an exact and simple formula valid for arbitrary spectra and thresholds. This formula will be obtained for an arbitrary process of ν_e disappearance (e.g. oscillation accompanying by the decay of a mass-eigenstate [10]), characterised by ν_e survival probability $P_{ee}(E)$. The oscillation, as most interesting and realistic case, will be analysed in detail later and anticipating it we shall keep the subscript "osc" (for oscillation) as a notation from the beginning.

The basic quantity that we shall analyse is the ratio of the electron rate in SK, $R^{SK}_{\nu_a}$, produced by active neutrinos $\nu_a = \nu_{\mu,\tau}$ and the total electron rate $R^{SK}_{\rm tot}$:

$$r_{\rm osc} = R^{SK}_{\nu_a}/R^{SK}_{\rm tot}. \tag{1}$$

Let us introduce the following definitions.

1. The exact flux of ν_e neutrinos reaching a detector is defined as

$$\Phi_{\nu_e}(E_\nu) = \Phi_B \phi_{SSM}(E_\nu) P_{ee}(E_\nu), \tag{2}$$

where Φ_B is the total boron neutrino flux at production, the value of which is not specified, and $\phi_{SSM}(E_\nu)$ is the spectrum at production, normalised as

$$\int \phi_{SSM}(E_\nu) dE_\nu = 1. \tag{3}$$

2. According to the definition of SK collaboration, the detected ^{8}B neutrino flux, ϕ^{ES}_{SK}, determines the total rate in SK above threshold T^{SK}_{th}, *if the neutrino spectrum in detector is standard*:

$$R^{SK}_{\rm tot} = \phi^{ES}_{SK} \int_{T^{SK}_{th}}^{T^{max}} dT \int dT' R_{SK}(T,T') \int_{E_\nu^{min}(T')}^{E_\nu^{max}(T')} dE_\nu \phi_{SSM}(E_\nu) \sigma_{\nu_e e}(E_\nu, T'), \tag{4}$$

where T' and T are the real and the measured kinetic energies of the electron, respectively, and $R_{SK}(T,T')$ is energy resolution function of the Super-Kamiokande detector [11]. ϕ^{ES}_{SK}

can be interpreted as the flux of ν_e neutrinos in case of $\nu_e \to \nu_s$ conversion, and as $\phi_{SK}^{ES} = \phi_{\nu_e} + 0.154\phi_{\nu_a}$ in case of $\nu_e \to \nu_s$ conversion, where 0.154 is a ratio $\sigma(\nu_a e)/\sigma(\nu_e e)$.

3. The flux of electron neutrinos from ^{8}B decays, ϕ_{SNO}^{CC}, is determined by SNO collaboration from the CC-reaction rate above threshold $T_{\rm th}^{SNO}$, *assuming the standard spectrum*:

$$R_{SNO}^{CC} = \phi_{SNO}^{CC} \int_{T_{\rm th}^{SNO}}^{T_{max}} dT \int dT' R_{SNO}(T,T') \int_{E_\nu^{min}(T')}^{E_\nu^{max}(T')} dE_\nu \phi_{SSM}(E_\nu)\sigma_{CC}(E_\nu, T'), \tag{5}$$

where $R_{SNO}(T,T')$ is energy resolution function of SNO [1].

4. On the other hand the same rate R_{SNO}^{CC} can be expressed via the exact flux $\Phi_{\nu_e}(E_\nu)$, as given by Eq.(2):

$$R_{SNO}^{CC} = \Phi_B \int_{T_{\rm th}^{SNO}}^{T_{max}} dT \int dT' R_{SNO}(T,T') \int_{E_\nu^{min}(T')}^{E_\nu^{max}(T')} dE_\nu \phi_{SSM}(E_\nu) P_{ee}(E_\nu)\sigma_{CC}(E_\nu, T'). \tag{6}$$

5. Finally, we define two quantities, J_i and $J_i\{P_{ee}\}$

$$J_i\{P_{ee}\} \equiv \int_{T_{\rm th}^{i}}^{T_{max}} dT \int dT' R_i(T,T') \int_{E_\nu^{min}(T')}^{E_\nu^{max}(T')} dE_\nu \phi_{SSM}(E_\nu) P_{ee}(E_\nu)\sigma_i(E_\nu, T'), \tag{7}$$

$$J_i \equiv J_i\{P_{ee} = 1\}, \tag{8}$$

for $i = SNO,\ SK,\ \ \sigma_i = \sigma_{CC}$ for SNO and $\sigma_i = \sigma_{\nu_e e}$ for SK. The ratio $J_i\{P_{ee}\}/J_i$ determines the average survival probability $\langle P_{ee}\rangle_i$ for $i =$SNO, SK. This ratio is P_{ee} in case of constant survival probability.

We are now ready to obtain the exact expression for the ratio $r_{\rm osc}$ from Eq.(1). Rearranging it as $r_{\rm osc} = 1 - R_{\nu_e}^{SK}/R_{\rm tot}^{SK}$, and using for SK rate induced by ν_e $\ R_{\nu_e}^{SK} = \Phi_B J_{SK}\{P_{ee}\}$ we obtain

$$r_{\rm osc} = 1 - \frac{\Phi_B J_{SK}\{P_{ee}\}}{\phi_{SK}^{ES} J_{SK}}, \tag{9}$$

where Φ_B is given by Eqs.(5) and (6) as

$$\Phi_B = \phi_{SNO}^{CC} \frac{J_{SNO}}{J_{SNO}\{P_{ee}\}}. \tag{10}$$

We can decompose $P_{ee}(E)$ in the energy independent part $\bar{P}_{ee}$, e.g. the average value in one of the experiments, and a small (according to experimental data) energy dependent part $\delta P(E)$: $\quad P_{ee}(E) = \bar{P}_{ee} + \delta P(E)$. Then we can re-write Eq.(9) as

$$r_{\rm osc} = 1 - \frac{\phi^{CC}_{SNO}}{\phi^{ES}_{SK}} \frac{1 + J_{SK}\{\frac{\delta P_{ee}}{\bar{P}_{ee}}\}/J_{SK}}{1 + J_{SNO}\{\frac{\delta P_{ee}}{\bar{P}_{ee}}\}/J_{SNO}} = 1 - \frac{\phi^{CC}_{SNO}}{\phi^{ES}_{SK}} \frac{1 + \langle\frac{\delta P_{ee}}{\bar{P}_{ee}}\rangle_{SK}}{1 + \langle\frac{\delta P_{ee}}{\bar{P}_{ee}}\rangle_{SNO}}, \tag{11}$$

where the angle brackets mean energy averaging. The formula (11) is exact. It is valid in particular even when $\delta P(E)$ is not small in comparison with $\bar{P}_{ee}$. It is derived for any process of ν_e disappearance, including the oscillation.

The correction factor in Eq.(11),

$$K_{\rm corr} = \frac{1 + \langle\frac{\delta P_{ee}}{\bar{P}_{ee}}\rangle_{SK}}{1 + \langle\frac{\delta P_{ee}}{\bar{P}_{ee}}\rangle_{SNO}}, \tag{12}$$

is close to 1 in each of two cases: when *(i)* $\delta P_{ee}/\bar{P}_{ee}$ is small or when *(ii)* average values $\langle \delta P_{ee}/\bar{P}_{ee}\rangle$ for SNO and SK are close to each other. As we will see in the physically relevant cases both conditions work simultaneously.

The first condition is satisfied because the observed distortion of spectrum is small. The correction factor (12) is invariant relative to decomposition $P_{ee}(E) = \bar{P}_{ee} + \delta P_{ee}(E)$ with arbitrary $\bar{P}_{ee}$. Since the distortion of spectrum is small we always can choose the value of $\bar{P}_{ee}$ such that $\delta P_{ee}(E)$ changes its sign in the middle of the observed energy interval. It provides the smallness of $\langle \delta P_{ee}/\bar{P}_{ee}\rangle_i$. The condition *(ii)* will further diminish $1 - K_{\rm corr}$.

In general case $K_{\rm corr}$ can be obtained directly from observational data using $\delta P_{ee}(E)$ for each energy bin.

We shall consider now the application of Eq.(11) for neutrino oscillations assuming that *oscillation is the only way of* ν_e *disappearance.*

To test the different hypotheses we shall treat $r_{\rm osc}$, given by Eq.(11), as experimental value to be compared with a theoretical value $r_{\rm osc} = 0$. The latter is the predicted value for the hypothesis of oscillation to sterile neutrino only, as in the case of two neutrino mixing (hereafter we shall refer to this case as *sterile-neutrino oscillation*). In the case of oscillation to active-neutrinos ν_a, to be referred to as *active-neutrino oscillation*, $r_{\rm osc} = 0$ corresponds to absence of $\nu_e \to \nu_a$ oscillation.

For the case $P_{ee}(E) = const$ we obtain

$$r_{\rm osc} = 1 - \frac{\phi^{CC}_{SNO}}{\phi^{ES}_{SK}} = 0.246 \pm 0.0694. \tag{13}$$

In particular, this value of the ratio is exact for no-oscillation case $P_{ee} = 1$, and formally it is $3.54\,\sigma$ away from no-oscillation value $r_{\rm osc} = 0$.

However, evaluation of the error for the ratio $\phi^{CC}_{SNO}/\phi^{ES}_{SK}$ does not correspond to to the usual connection with the confidence level, because the distribution is not Gaussian. More correct evaluation of the error is as follows.

In the plane $(\phi^{CC}_{SNO}\ ,\ \phi^{ES}_{SK})$ one plots experimentally measured point and $1\,\sigma$, $2\,\sigma$ and $3\,\sigma$ contours around it (see Fig. 1). One can see that the contour $3.33\,\sigma$ touches the line $r_{\rm osc} = 0$.

In the general case to obtain the value $r_{\rm osc}$ from Eq.(11) we evaluate $K_{\rm corr}$ in the following way.

First we assume, and later verify, that correction factor for a tested hypothesis is given by value $K_{\rm corr} \sim 1$ within very narrow interval $\Delta K_{\rm corr}$, so that we can treat $K_{\rm corr}$ just as a number. Then using $\phi^{CC}_{SNO}/\phi^{ES}_{SK} = 0.754 \pm 0.0694$ we find formally for this hypothesis

$$r_{\rm osc} = 1 - 0.754\, K_{\rm corr} \pm 0.0694 K_{\rm corr} \tag{14}$$

To obtain the range of allowed values of $K_{\rm corr}$ one must compute the correction factors for all points in the parameter oscillation space allowed by all available data e.g. at 1σ, with the Super-Kamiokande spectra data being the most important ingredient.

If the starting assumption about narrow interval $\Delta K_{\rm corr}$ is confirmed, one can proceed further choosing the maximum value of $K_{\rm corr}$ from obtained range and inserting it to Eq.(14). $K^{\rm max}_{\rm corr}$ gives the smallest value of $r_{\rm osc}$ with the largest error, which we conservatively take as the final result.

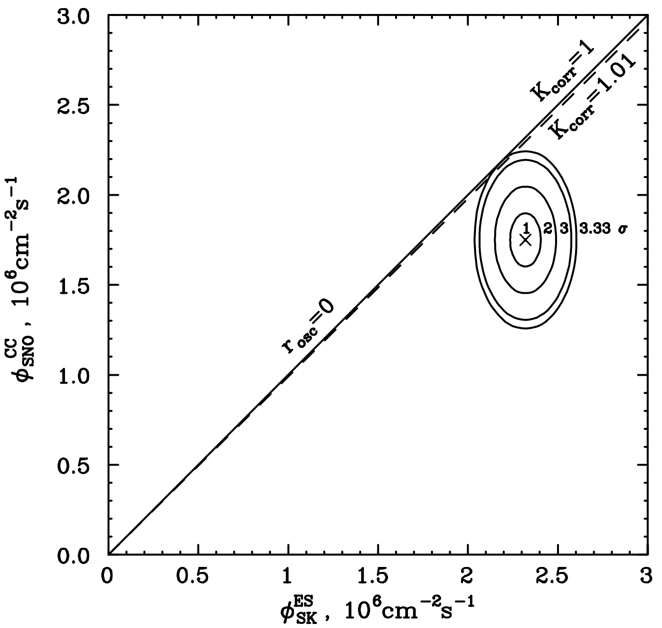

FIG. 1. SNO and Super-Kamiokande fluxes allowed at 1 σ, 2 σ and 3.33 σ levels in comparison with no-oscillation value $r_{\rm osc} = 0$ for two values of correction factor $K_{\rm corr} = 1$ and $K_{\rm corr} = 1.01$. The experimental point is 3.3 σ away from $r_{\rm osc} = 0$ which corresponds to absence of active neutrino oscillation.

In both cases, sterile-neutrino oscillation and active-neutrino oscillation, a statistically significant deflection from $r_{\rm osc} = 0$ is a proof of the hypothesis (absence of sterile-neutrino oscillation, and presence of active-neutrino oscillation), though for sterile neutrinos this criterion is formally more strict, because $r_{\rm osc} = 0$ is the theoretical prediction of sterile-neutrino hypothesis.

For sterile neutrinos $K_{\rm corr}$ has been calculated [1] for all points in the sterile oscillation parameter space $(\Delta m^2, \tan^2\theta)$ allowed by the Super-Kamiokande event rates and the day-night spectra at 1σ. It results in $0.97 \leq K_{\rm corr} \leq 1.03$, which, as expected, does not differ much from the energy-constant value 1. Using Eq.(14) with $K_{\rm corr}^{\rm max} = 1.03$ one obtains that the sterile oscillation is excluded with formal significance $\geq 3.1\,\sigma$. For the realistic evaluation significance is less (see Fig.1).

If to add to the Super-Kamiokande data all other pre-SNO data (including the measured rates in the chlorine and gallium experiments), the status of sterile-neutrino oscillation becomes further disfavoured. In this case the range of allowed values of $K_{\rm corr}$ is slightly shifted to lower values $0.93 \leq K_{\rm corr} \leq 1$. With this range, the sterile-neutrino oscillation is excluded at significance better than $3.3\,\sigma$, as it can be seen from Fig.1.

For active neutrino-oscillation the analysis can be made in identical way. In this case the correction factors have to be calculated for all points in the parameter space $(\Delta m^2, \tan^2\theta)$ allowed by all experimental data. In fact we used the range of calculated values of $K_{\rm corr}$ for the points of best fit solutions [12] only (see Table 1) as $0.944 \leq K_{\rm corr} \leq 1.002$. From Eq.(14) we obtain formally $r_{\rm osc} = 0.244 \pm 0.0695$. From Fig.1 we see that that observed fluxes for this range of correction factors are 3.3σ away from $r_{\rm osc} = 0$, which corresponds to hypothesis about absence of active-neutrino oscillation.

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[1]The analysis of sterile-neutrino oscillation, including the calculations of $K_{\rm corr}$ has been performed by M.C.Gonzalez-Garcia and C.Pena-Garay. They have also calculated the data of Table 1.

TABLE I. Correction factors $K_{\rm corr}$ for the best fit solutions

solution	$\tan^2\theta$	Δm^2, eV^2	$K_{\rm corr}$
LMA	0.365	$3.65 \cdot 10^{-5}$	1.002
SMA	0.00061	$5.01 \cdot 10^{-6}$	0.944
LOW	0.708	$1.03 \cdot 10^{-7}$	0.993
JS^2	1.000	$5.46 \cdot 10^{-11}$	0.995

In conclusion, we have derived the exact formula (11) for calculation of ratio of the rate in Super-Kamiokande from $\nu_{\mu,\tau}$ neutrinos to the total rate. This ratio, $r_{\rm osc}$, characterises the oscillation of electron neutrino to another active neutrino. The formula is valid for arbitrary thresholds in SNO and SK experiments, and arbitrary neutrino spectra. In particular, smallness of $\delta P_{ee}/\bar{P}_{ee}$ is not required for validity of Eq.(11). We have shown that this ratio can be used to test the hypotheses of no-oscillations and oscillations to sterile and active neutrinos.

For the no-oscillation case the ratio $r_{\rm osc}$ is given by Eq.(13) and this case is excluded at $3.3\,\sigma$ by comparison with theoretical no-oscillation value $r_{\rm osc} = 0$.

For a small energy-dependent component of survival probability, $\delta P_{ee}/\bar{P}_{ee}$, the ratio is described with a good accuracy by the same Eq.(13), which would exclude $\nu_e \to \nu_s$ oscillation at 3.3σ. This hypothesis is further tested evaluating numerically the correction factor $K_{\rm corr}$ for all points in the parameter space $(\Delta m^2, \tan^2\theta)$ in the pre-SNO allowed region for sterile-neutrino oscillations. It is found that sterile-neutrino oscillation is excluded at 3.3σ.

For oscillation to active neutrinos our analysis is not as accurate as for sterile neutrinos. The correction factors are calculated only for the best-fit points in the parameter space. With these data the oscillations to active neutrino is confirmed at 3.3σ.

With statistical error reduced by factor 2, the the significance of above results will reach 3.5σ.

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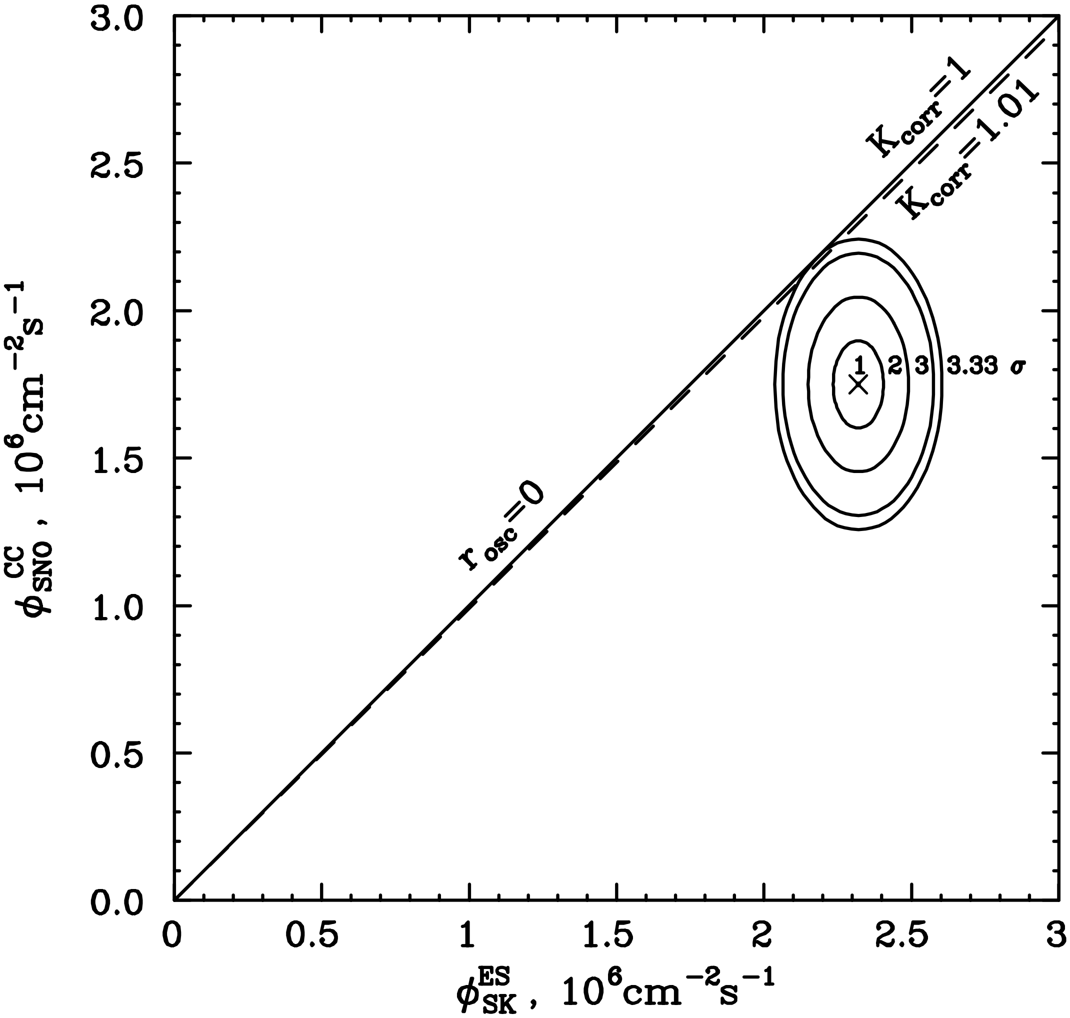

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$K_{corr}=1.01$
1 2 3 3.33 σ
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ϕ^{CC}_{SNO} , $10^6 cm^{-2} s^{-1}$
ϕ^{ES}_{SK} , $10^6 cm^{-2} s^{-1}$